Informally, a set is a collection of things:

\[ A = \{\text{Alex, Tippy, Shells, Shadow}\} \quad \text{Pet names} \]

\[ B = \{\text{red, blue, yellow}\} \quad \text{Primary colors} \]

\[ C = \{\{a, b\}, \{b, c\}, \{a, d\}, \{a, b, c\}\} \quad \text{A set of sets} \]
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- The order of elements is irrelevant — a set is an unordered collection
- There are no repeated elements in a set
Infinite Sets and Set Builder Notation

Here is an example of an infinite set:

\[ T ::= \{0, 3, 6, 9, 12, \ldots\} \]
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We can state this more precisely as

\[ T ::= \{x \in \mathbb{N} \mid x \text{ is a multiple of } 3\} \]
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We can state this more precisely as

\[ T ::= \{x \in \mathbb{N}|x \text{ is a multiple of } 3\} \]

or more compactly as:

\[ T ::= \{3n|n \in \mathbb{N}\} \]
## Predefined Sets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\emptyset$</td>
<td>The empty set</td>
</tr>
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<td>N</td>
<td>The set of natural numbers, i.e., ${0, 1, 2, 3, \ldots}$</td>
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<td>Z</td>
<td>The set of integers, i.e., ${0, -1, 1, -2, 2, \ldots}$</td>
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<td>$\subset$</td>
<td>$A \subset B \text{ iff } A \subseteq B \text{ and } A \neq B$</td>
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Showing Two Sets $A$ and $B$ are Equal

- Show that $x \in A$ implies $x \in B$ and vice-versa
- Show $A \subseteq B$ and $B \subseteq A$
If all the sets being considered are a subset of a larger set \( U \), we call that a *universal* set.
Universal Set and Complement

- If all the sets being considered are a subset of a larger set \( U \), we call that a *universal* set.

- In the presence of a universal set, we can define a *complement* \( \overline{A} \) as follows:

\[
\overline{A} = U - A
\]

Alternatively:

\[
\overline{A} = \{ x \in U | x \notin A \}
\]
Some properties of Set Operators

- $\cup$ and $\cap$ are commutative and associative

  - Follows from the definition of these operators, and the fact that the boolean connectives $\text{or}$ and $\text{and}$ are both associative and commutative

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Can be established from the definition of $\cup$ and $\cap$

De Morgan's Law

$A \cup B = \overline{(A \cap B)}$

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Some properties of Set Operators

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- De Morgan’s Law
  \[
  \overline{A \cup B} = \overline{A} \cap \overline{B} \\
  \overline{A \cap B} = \overline{A} \cup \overline{B}
  \]
Given a set $A$, the *power set* of $A$, denoted $\mathcal{P}(A)$ is

$$\mathcal{P}(A) = \{x | x \subseteq A\}$$

- A powerset always includes $\emptyset$
- $\mathcal{P}(A)$ always includes $A$
If the size of $A$, denoted $|A|$, is $n$, what is the size of $\wp(A)$?
Products and Tuples

**Cartesian Product of sets A and B:** \( A \times B = \{(a, b) | a \in A, b \in B\} \)

**Example:** Let \( A = \{1, 4, 9\} \), \( B = \{a, e, i, o, u\} \)
**Products and Tuples**

*Cartesian Product of sets A and B:* \[ A \times B = \{(a, b)|a \in A, b \in B\} \]

**Example:** Let \( A = \{1, 4, 9\} \), \( B = \{a, e, i, o, u\} \)

\[
A \times B = \{(1, a), (1, e), (1, i), (1, o), (1, u), \\
(4, a), (4, e), (4, i), (4, o), (4, u), \\
(9, a), (9, e), (9, i), (9, o), (9, u)\}
\]
(x, y) is called an (ordered) pair
(x, y, z) is a triple
More generally, (x_1, x_2, \ldots, x_n) is an \(n\)-tuple, or just a tuple
Unlike sets, the order of components is important in a tuple. \(A \times B \neq B \times A\)
\(A^n\) is short hand for
\[
\underbrace{A \times A \times \cdots \times A}_{n \text{ times}}
\]
Sets: Summary

- Definition of sets
- Set builder notation
- Set operators: membership, subset, union, intersection, difference
  - Properties of set operators: commutativity, associativity, distributivity
- Set equality
- Universal set, set complement and De Morgan’s Laws
- Power Set
- Cartesian Product and Tuples