## Predicates (Textbook §3.6)

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#### Predicates (Textbook §3.6)

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$$P(n) ::=$$
 "n is a perfect square"
$$P_{3} \underbrace{4,5} T$$

$$P_{4} \underbrace{2,3,4} F$$

- If P is a unary predicate, P(n), being a proposition, is either true or false.
- A k-ary predicate takes k variables, e.g.,

$$Py(x, y, z) := "x^2 + y^2 = z^2 \text{ for integers } x, y \text{ and } z$$
"

is a ternary predicate that characterizes Pythagorean triples. x, y and z are said to be *free variables* in the formula defining Py.

$$Q(x,y) := x \ge y \vee y > x$$

## Satisfiability and Validity

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- Related to *existential* quantification
- Example: Are there values of x, y and z such that Py(x, y, z)?

$$R(x) := x/x$$

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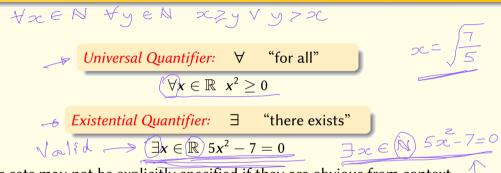
Valid: The formula is true for all possible values of variables

• Related to *universal* quantification.

Example: 
$$Q(x, y) ::= \text{``if } y > 0 \text{ then } x + y > x \text{''}$$

Is this always true?

#### Quantifiers



The sets may not be explicitly specified if they are obvious from context.

Unsat

If multiple variables belong to the same domain, we may abbreviate:

tayen xzy vy>x

#### Free Vs Bound Variables

 A quantifier "captures" or "binds" a variable. It is no longer free to take any possible value

 $P(n) := \exists x \in \mathbb{N}, \ n = \underline{x^2}$ 

- This formula has two variables *n* and *x*, but *x* has been bound by the quantifier.
- *n* continues to be *free*: It can take any value.
  - Some values of n make P(n) true, while others render it false.

# **Examples of Quantified Statements**

"Every American has a dream."

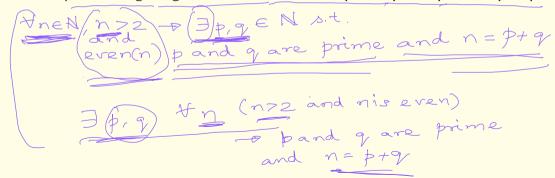
## **Examples of Quantified Statements**

Every even integer greater than 2 is the sum of two primes

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Every even integer greater than 2 is the sum of two primes

• For every even integer n greater than 2, there exist primes p and q s.t. n = p + q



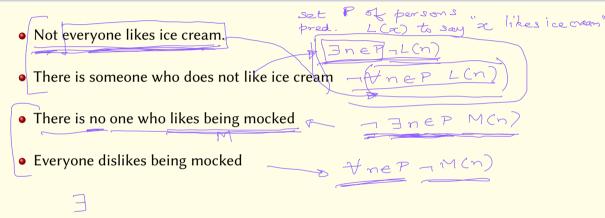
#### Order of Quantifiers

- Sometimes the order does not matter
  - e.g., when two universal quantifiers are nested

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- But other times, they mean a lot! One order will make sense but the other will not.
  - Often, you cannot change the order of existential operators

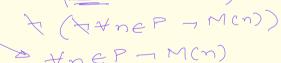
# **Negating Quantifiers**



# **Negating Quantifiers**

Not everyone likes ice cream.

- T(NEP LCN)
- There is someone who does not like ice cream
- There is no one who likes being mocked → (∃ne P M(n))
- Everyone dislikes being mocked
- ∀ is equivalent to ¬∃¬
- $\exists$  is equivalent to  $\neg \forall \neg$



## **Unit Summary**

- Predicate Vs Proposition
- Satisfiability and Validity
- Quantifiers
  - Free and Bound variables
  - Conversion between English and Logical Formulas
- Order of quantifiers
- Negating quantifiers